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EXPERIMENTAL STUDY OF SIMPLE STRUCTURE

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A battery of thirty-six tests was given to a group of high-school seniors. The factorial analysis reveals essentially the same primary factors that were found in previous studies. The test battery reveals a simple structure.

In two previous factorial studies of large batteries of psychological tests, simple structure was found in the rotated reference frame (1, 2). The factorial experiment to be reported here constitutes a third investigation of a large test battery with regard to the existence of simple structure and the psychological interpretation of the rotated reference axes. As a secondary problem we devised eight new tests which were inductive in character, according to a previous tentative interpretation of this factor. These new tests were included in the battery in order to determine whether they had a factor in common which would sustain the tentative interpretation of the inductive factor. Most of the tests were revised forms of previous tests in that the new battery was arranged for machine scoring. Some of the tests were only slightly altered for this purpose, while other tests were altered considerably. In revising the tests for machine scoring the previous names have been retained for convenience in identifying the nature of the test content. The problem whether the factorial composition of a test is invariant when it is moved from one test battery to another, or when the tests are given to comparable populations, must be answered in terms of future experiments in which the same tests will be incorporated in different test batteries for the same population and for comparable populations.

In the first large test battery to be investigated by the factor methods (1), the experimental population was a group of 240 volunteer students at the University of Chicago. They constituted a highly selected group. Our second study (2) of this kind was done at the Lane Technical High School on a class of several hundred seniors. The present study was made with 286 seniors at the Hyde Park High School in Chicago. The tests were given on five consecutive days, October 18 to 22, 1937. This population was different from the previous experimental populations in that it represented more nearly the students found in high schools with a general curriculum and in which

a fairly large proportion of the graduates continue their education in one of the professions.

In Table 1 we have a list of the names of the tests and their saturations on the primary reference axes. The tests were given in two parts, a fore-exercise with appropriate instructions, and the test proper. These parts were given with separate time limits. The time limits are listed in Table 1; the first entry is the number of minutes for the fore-exercises, the second the number of minutes for the test proper. The fore-exercise time was regarded as flexible. Each of the memory tests had four time limits, the first two for the fore-exercise, the third for study time, and the fourth for recall. Test 30 required no fore-exercise.

We shall describe very briefly the nature of each test. For the reader who may want the complete set of the tests, we have filed with the American Documentation Institute, in Washington, D. C., a film record of the complete test battery, including instructions and fore-exercises (3). The availability of these film records makes it unnecessary to reproduce in print the whole set of tests.

Except for adaptation to machine scoring, the content of a large number of the tests was the same as in previous tests of the same names. These tests were: 1, Addition; 2, Areas; 3, Arithmetic; 4, Cards; 5, Completion; 7, Disarranged Words; 10, Identical Forms; 11, Identical Numbers; 12, Initials Recall; 16, Mechanical Movements; 18, Multiplication; 20, Number Series; 22, Proverbs; 23, Pursuit; 28, Same-Opposite; 30, Spelling; 31, Squares; 32, Verbal Analogies; 33, Verbal Enumeration; 34, Word Number.

In assembling the new battery we retained two or three tests for each of the primary abilities that had been found in previous studies. To these were added certain new tests which were included for the purpose of investigating the several factors. The number factor N was represented by 1, Addition; 3, Arithmetic; 18, Multiplication; 19, Number Patterns; and 20, Number Series. The only new test in this list is Number Patterns. In this test the subject is shown a square with five rows and five columns. He is asked to discover the rule which determines the spatial arrangement of the digits and to determine the particular digit which belongs in a given cell. In the instructions and fore-exercise it is explained that the digits are arranged in consecutive order in all of the problems, that the sequence may run from 0 to 9 or parts of this range, and that the problems differ only in the spatial arrangement of the numerical sequence in the squares. This test was devised as a test of induction which is numerical in content. The inductive character of the test is in the nature of the task, to discover, for each square, the spatial arrangement of the sequences of

digits so that the missing number X can be determined.

The verbal factor V was represented in 5, Completion; 6, Directions; 22, Proverbs; 28, Same-Opposite; and 35, Word Patterns. The Directions test consisted of short instructions to be carried out by the subject on the test form. It was similar to a test of the same name by Woodworth and Wells.

The space factor S was represented by 4, Cards; 9, Figures; and 31, Squares, which were improved forms of previous tests for this factor.

The memory factor M was represented by 12, Initials Recall; and 34, Word Number. Both of these tests have been used previously in hand-scoring form.

The perceptual factor P was represented by 10, Identical Forms; 11, Identical Numbers; 27, Repeated Letters; 29, Scattered X's; and 33, Verbal Enumeration. These tests are similar to tests previously used. In Repeated Letters the subject ringed all letter combinations in which two or three adjacent letters were identical. The test form consisted of several pages of pied letters.

The word factor W was represented by 7, Disarranged Words; and 17, Mirror Reading. In both of these tests the subject was asked to extract words from disarranged letters.

The inductive factor I was represented in this test battery by several new tests. The tests for this factor were as follows: 8, Figure Grouping; 13, Letter Grouping; 14, Letter Series; 15, Marks; 19, Number Patterns; 20, Number Series; 21, Patterns; 35, Word Patterns. Figure Grouping is an adaptation of Spearman's Figure Classification test that was used in an earlier study (1). In Letter Grouping the subject is shown four groups of letters with five letters in each group. Three of the groups have something in common, and the subject is asked to mark the odd group. This test was designed by Mr. Herbert Landahl. In Letter Series the subject is shown a series of letters such as *aabccdeef*, and he is asked to write the next letter in the series. The test is arranged in increasing order of difficulty. It was designed by Thelma Gwinn Thurstone. The test called Marks was a paper-and-pencil form of the Yerkes multiple-choice test. Number Patterns was described in a previous paragraph. The test called Patterns involved the discovery of repetitions in a pattern of rectangular parts such as are used in some linoleum rugs. Word Patterns was a test of induction on columns of words. It was designed by Miss Leone Chesire.

Deduction was represented by three syllogistic tests that varied somewhat in content and form. Two of these tests were adapted from reasoning tests by Cyril Burt. In addition to these deductive tests we

have the test in arithmetical reasoning with statement problems, which has been found previously to be deductive in character. The Mechanical Movements test is deductive in form but fundamentally different from the other deductive tests in content. The other deductive tests are all verbal. Verbal Analogies is deductive, and the Number Series test involves both induction in the discovery of the rule and deduction in checking the rule in the answers.

Pearson product-moment correlations were determined for all pairs of tests. The product-moment correlations are shown in Table 2. The centroid matrix with eleven factors is shown in Table 3, and the frequency distribution of eleventh factor residuals in Figure 1. The

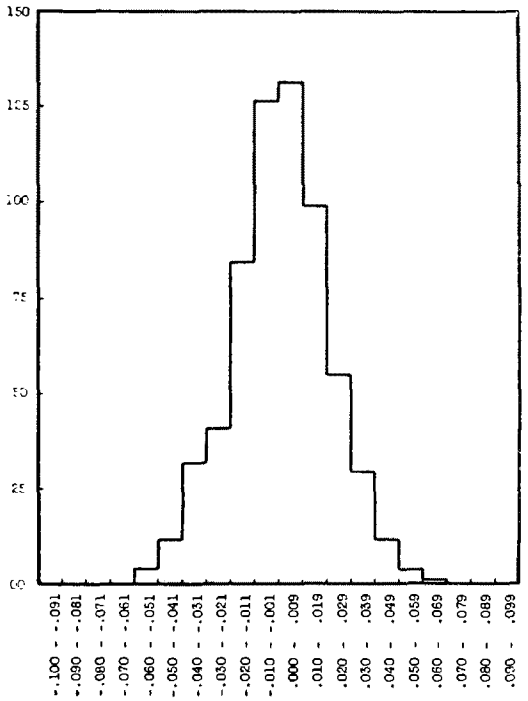

Figure 1

projections of the test vectors on the rotated reference frame are also shown in Table 1. In this table eight primary factors have been indicated in accordance with the nature of the tests which have large saturations on each primary. Three residual planes 9, 10, and 11 are left without interpretation.

The matrix of the transformation Λ from the centroid matrix to the rotated matrix of primary reference axes is given in Table 4, and

the intercorrelations of the primary reference factors are shown in Table 5.

In Table 1 the first column contains 22 factor loadings that are in the range ±.10, and it contains three entries that are higher than .30. The three tests and their saturations on this factor are Identical Forms (.58), Scattered X's (.35), and Verbal Enumeration (.54). These are tests which have previously been interpreted as representative of the perceptual factor *P*.

The second column contains 29 nearly vanishing entries in the range ±.10, and it has three tests with saturations greater than .30. These are Addition (.54), Arithmetic (.38), and Multiplication (.62). Variable No. 37, Sex, has a negative loading of —.31 on this factor which indicates that the boys did better in the numerical tests than the girls. Two other tests had saturations near .30, namely, Number Patterns (.29) and Number Series (.27). These findings agree with the results of previous factor experiments in that the simple number tests such as Addition and Multiplication have the highest saturations on the number factor *N*, while the more complex tests involving numerical content have lower saturations on this factor. There can hardly be any question about the identification of this factor.

The column *W* has 21 nearly vanishing factor loadings, and it has or possibly three, significant loadings. The tests are Disarranged Words (.42), Mirror Reading (.42), Letter Series (.34), and Number Patterns (.30). This factor is tentatively identified by Disarranged Words. The reading of words in a reversed position, as in a mirror, has something in common with the deciphering of words in which the letters are presented in random order. The psychological nature of this factor should be investigated in order to determine its fundamental nature. In a previous factor experiment the Disarranged Words also showed high saturation on a factor which is independent of the verbal factor *V*.

The column *V* has nearly 21 vanishing factor loadings, and it has significant loadings on the following tests: Completion (.58), Directions (.42), Proverbs (.65), Same-Opposite (.68), Verbal Analogies (.41), Word Patterns (.46). There are lower but possibly significant saturations on the following: Letter Grouping (.31), the three syllogism tests (.34, .26, .35), Spelling (.36). This is clearly the same factor which has been identified in previous experiments as the verbal factor *V*. Its identification seems to be as clear as that of the number factor.

The column *S* has 31 nearly vanishing entries. The following tests have significant factor loadings on this factor: Cards (.63), Figures (.69), Squares (.42). This is the space factor *S* which has

been found in previous experiments. In previous studies of these tests the Pursuit test has involved different saturations in the space factor and the perceptual factor. In simplifying a test there seems to be a tendency for the factorial composition to become more perceptual in character. Whether this implies a shift toward some form of speed factor with the simplification of a test cannot be determined from data so far available, but this is a possible interpretation. Factorial study of speed of performance of simple tasks might reveal not only the relation of one or more speed factors to the primaries here discussed, but also the nature of the perceptual factor.

The column *M* has vanishing projections on all the tests except the two memory tests which were included in this battery, namely, Initials Recall (.59) and Word Number (.58).

The column *I* has no saturations so high as those which identify the number, verbal, space, and memory factors. This column has 19 vanishing projections. Listing all the saturations above .25 we have the following: Directions (.29), Figure Grouping (.31), Letter Grouping (.36), Letter Series (.39), Marks (.43), Number Patterns (.39), Number Series (.26), Patterns (.26), Pursuit (.26), and Squares (.27). The Word Patterns test has a loading of .24. No one test has a high saturation on the factor *I*, but it is probably significant that all of the eight tests that were specially designed to involve inductive thinking are included in the list. None of the specially designed inductive tests has zero saturation on this factor. All but three of the above tests were included in the battery for the specific purpose of representing the inductive factor. The three exceptions are Directions, which was a new test of unknown factorial composition, the Pursuit test, which has been erratic before as regards the space factor and the perceptual factor, and Squares, which was designed to be a test of the space factor in which it does have a higher saturation, namely, .42. Pursuit and Squares are both tests which can be done in at least two ways, one way being faster than the other. The discovery of the best way to do a test might be responsible for a small inductive component. According to these results the Yerkes multiple-choice test, which is here called Marks in a paper-and-pencil form, is the best test for the inductive factor. The next best tests for this factor would seem to be Letter Series and Number Patterns. The tests in this list have, on the average, about ten per cent of their total variance attributable to the inductive factor. It is a question for further experimental study to determine whether tests can be devised for this factor which have higher saturations. Any test for this factor may be subject to the limitation that an inductive task requires some form

of content which may involve other primaries. Hence, the factorial composition of an inductive test may be necessarily more complex than tests for the other primary factors. This difficulty may be overcome by the discovery of some measurable parameters which represent this factor more directly than the performance of a complex inductive task.

The column *D* has 20 nearly vanishing entries. The tests with significant factor loadings in this column are Arithmetical Reasoning (.49), Mechanical Movements (.46), Number Series (.47), Verbal Analogies (.32), and the three syllogism tests (.27, .38, .36). This factor has been identified before as deductive in character. Number Series has saturation in both the inductive and deductive factors.

Column 9 has only three or four tests with significant saturations, namely, Identical Numbers (.46), Scattered X's (.44), and possibly Repeated Letters (.29), and Squares (.25). All other entries in this column are vanishingly small. All of these tests have in common that the subject hunts over the page, or through a column or a row, for some identity. This feature is prominent in the first three of the four tests listed. This characteristic of column 9 might be used as a basis for investigating a larger number of tasks with similar features in order to determine whether any clear primary factors can be found in them.

The two remaining columns do not show any large saturations, and they are consequently left as residual factors.

In order to determine the correlations between the primary factors, we turn to the matrix Λ_{mp} of Table 4. The columns of this matrix show the direction cosines λ_{mp} of the primary reference vectors Λ_p. The direction cosines t_{mp} of the primary trait vectors T_p are proportional to the entries in rows of $\Lambda^{-1}{}_{mp}$. In Table 5 we have the cosines of the angular separations between the reference vectors Λ_p. The cosines are given by the matrix product $\Lambda'\Lambda$.

Table 6 shows the cosines of the angular separations of the primary trait vectors T_p. Since these are unit vectors, the cosines are also their intercorrelations. Most of the correlations are low, but some of them are appreciable. The perceptual factor seems to be quite independent of the other primary factors found so far except *W*. The number factor correlates higher than we should expect with the two verbal factors and with space and memory. The word factor has appreciable correlations with most of the other primaries as found in the present experiment. The matrix of Table 6 has been investigated to determine whether it can be interpreted as being essentially of rank one. If such an interpretation can be justified, we should be able to account for the correlations of the primaries by a single general

factor which might be the general factor postulated by Spearman. A best fitting single factor has been determined by a formula of Spearman (4), and we have listed the residuals in Table 7. These residuals are small. The largest residuals are found with the deductive factor, but this factor is unstable and not yet so clearly indicated as the others. There is some justification for interpreting these results as indicative of a second-order general factor which accounts for most of the correlations between the primary factors.

In Table 8 we have listed the saturation of each of the primary factors with the second-order general factor. It seems strange that induction is not represented. These saturations for the second-order general factor are quite different from those which have been recently determined for eighth-grade children in another factorial study. The correlations between the primary factors cannot yet be determined with the stability that is desirable for the investigation of second-order general factors.

Our principal findings in this study can be summarized as follows:

1) Primary factors have reappeared in several independent studies. They represent distinct functional unities. The primary abilities about which we now have a good deal of confidence are: (1) the number factor N, (2) the verbal factor V, (3) the word factor W, (4) the space factor S, and (5) the memorizing factor M. It should be distinctly understood that the isolation of primary functional unities of mind does not imply that these functions are indivisible. They are linearly independent functions, but they are not necessarily statistically independent, or uncorrelated.

2) The factors that reappear in successive studies but whose psychological nature has not yet been identified satisfactorily are: (1) the perceptual factor P, (2) the inductive factor I, and (3) a deductive or restrictive thinking factor D. That some functional unities of this kind exist seems clear, but we do not yet have a sufficient understanding of them to predict their behavior with certainty. To be sure, in the present investigation we constructed eight new tests for induction, and they all showed some variance on the same factor, but the saturations are not so high as we have obtained for other factors.

In response to many inquiries about these investigations we made available an experimental test battery for most of these factors. Our first determinations of the intercorrelations between the primary factors were made by taking the inverse of a reduced matrix Λ, omitting some of the dimensions of the common factors. This was an error which we have since corrected by using the complete matrix Λ. The result of the correction is to increase the correlations between the pri-

mary factors. The composites for the primary factors have still higher correlations because there are not available any pure tests for the primary factors. This limitation is universal in psychological and educational tests which usually overlap in common elements that are not represented in the criteria. In devising tests for a primary ability, it is the primary factor that serves as a criterion. A perfect test of a primary factor should have zero saturation on all but one of the primary reference traits, its specificity should vanish to assure the absence of unknown factors, and its reliability should be high. As long as the specificity of a test is appreciable, it involves unknown factors. So far we have not succeeded in devising tests with more than half of the total variance on a single primary factor, but this corresponds to a validity-correlation of about .70, which is quite satisfactory according to customary test standards. The experimental test battery should not be used as a service instrument. It is intended for those who want to experiment with tests for the primary mental abilities. A new battery for eighth-grade children has recently been completed with a number of test improvements which will be described in a separate publication.

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3. A complete set of the psychological tests used in this investigation, together with instructions and fore-exercises, is available as an Auxiliary Publication of the American Documentation Institute, Care of Science Service, 2101 Constitution Avenue, Washington, D. C. The test material is available in microfilm form as Document No. 1329 at a cost of $1.60. The film can be used in a 35 mm. film projector.
4. Thurstone, L. L. The vectors of mind. Chicago: Univ. Chicago Press, 1935. Equation (40), page 146.

TABLE 1

Rotated Factorial Matrix

TESTS	TIME LIMITS	*P*	*N*	*W*	*V*	*S*	*M*	*I*	*D*	9	10	11
1. Addition	1-7	—.034	.542	.086	—.043	—.025	—.014	.074	—.015	—.047	.034	—.187
2. Areas	3-8	.202	.065	.275	—.050	—.005	.089	.136	.160	—.013	—.067	—.005
3. Arithmetic	2-20	.027	.384	—.019	.057	.005	.084	.125	.489	—.035	—.066	.155
4. Cards	6-12	.072	—.038	.011	—.040	.630	.024	.020	.190	—.072	.011	—.065
5. Completion	2-5	.180	—.013	.069	.580	.088	—.011	—.070	.094	—.059	—.105	.075
6. Directions	2-7	.068	.059	.156	.420	—.041	.055	.293	.231	—.018	.126	.045
7. Disarranged Words	2-8	.030	.037	.421	.235	—.053	—.036	—.004	—.028	.080	.027	.056
8. Figure Grouping	5-6	.108	.099	.190	.004	.097	.006	.310	—.011	—.056	.162	—.103
9. Figures	5-11	—.005	—.027	.008	.072	.694	—.050	.057	.025	.027	.033	—.075
10. Identical Forms	3-8	.582	—.036	.209	.014	.050	—.036	.170	—.036	.007	—.010	.182
11. Identical Numbers	1-3	.289	.050	.007	.083	—.034	.043	.011	—.100	.464	.182	.159
12. Initials Recall	3-2-7-7	—.019	—.045	.038	.018	—.049	.588	—.022	—.024	—.008	—.079	.176
13. Letter Grouping	10-8	.066	.005	.256	.309	.012	—.076	.356	—.080	.097	.064	—.150
14. Letter Series	4-9	.107	.074	.336	.163	—.061	.071	.386	.164	—.032	.221	—.018
15. Marks	13-8	—.030	.038	.269	.051	.021	.077	.430	—.044	.001	—.071	.092
16. Mechanical Movements	9-15	.054	—.014	—.020	—.045	.165	.021	—.053	.465	.024	—.239	—.092
17. Mirror Reading	2-7	—.020	.009	.423	.061	.020	.075	.050	.184	.018	—.018	.076
18. Multiplication	1-7	.089	.617	—.011	.078	—.013	.001	—.048	—.042	—.020	—.036	—.024
19. Number Patterns	6-8	.073	.291	.297	—.037	.092	—.026	.392	.050	—.080	—.044	.271
20. Number Series	4-10	.114	.273	.074	.056	—.020	—.053	.256	.467	.040	.094	—.026
21. Patterns	3-4	.119	—.065	.289	—.019	.100	.035	.262	.157	.022	.256	.108
22. Proverbs	2-3	.033	—.065	—.068	.652	.051	.126	.185	.017	—.011	.038	—.078
23. Pursuit	2-7	.283	.162	—.041	—.045	.205	.081	.262	.197	.073	—.029	—.009
24. Reasoning I	2-5	—.059	.004	—.023	.344	—.049	.078	.214	.272	.072	.003	.099
25. Reasoning II	3-7	—.102	—.040	.036	.258	—.064	.055	.023	.376	—.011	—.061	—.126
26. Reasoning III	2-7	.135	—.009	.032	.349	.054	—.057	.089	.359	—.017	.256	.003
27. Repeated Letters	2-8	.280	—.002	.139	.017	.080	.062	.091	—.070	.294	.171	—.061
28. Same—Opposite (R)	2-5	.127	.056	.053	.679	.056	—.010	—.002	.006	—.039	.040	.215
29. Scattered X's	2-10	.346	—.042	.083	—.013	.032	—.078	—.080	.070	.436	—.034	.066
30. Spelling	10	—.068	.068	.179	.364	—.087	.063	—.094	.054	.033	.280	.153
31. Squares	2-6	—.018	.029	.009	—.022	.416	—.010	.266	.182	.248	—.057	.047
32. Verbal Analogies	2-7	.014	—.074	.215	.408	.026	—.073	.124	.320	—.090	—.054	—.037
33. Verbal Enumeration	2-6	.542	.046	.019	.200	—.052	.026	—.034	—.042	—.002	—.017	.196
34. Word Number	3-2-8-5	.022	—.034	—.023	—.046	.006	.579	.035	—.043	—.005	.065	—.092
35. Word Patterns	5-10	.102	—.095	.172	.455	—.067	.042	.244	.017	.049	—.043	.000
—36. Same—Opposite (W)	2-5	—.216	—.216	.085	.394	—.023	—.044	—.040	.248	.058	.065	—.034
37. Sex		.005	—.308	.175	.186	—.144	.186	.269	—.412	.014	.307	.079

TABLE 2

Correlation Matrix

	1	2	3	4	5	6	7	8	9	10	11	12	13
1		.220	.284	.072	.090	.224	.264	.219	.177	.029	.288	.078	.285
2	.220		.365	.339	.319	.380	.341	.250	.294	.332	.190	.187	.345
3	.284	.365		.358	.429	.527	.340	.224	.332	.149	.096	.228	.277
4	.072	.339	.358		.301	.298	.238	.264	.727	.304	.052	.092	.247
5	.090	.319	.429	.301		.588	.443	.175	.314	.327	.130	.249	.351
6	.224	.380	.527	.298	.588		.425	.259	.280	.277	.190	.218	.479
7	.264	.341	.340	.238	.443	.425		.252	.314	.318	.368	.237	.414
8	.219	.250	.224	.264	.175	.259	.252		.298	.276	.134	.089	.346
9	.177	.294	.332	.727	.314	.280	.314	.298		.305	.099	.068	.294
10	.029	.332	.149	.304	.327	.277	.318	.276	.305		.342	.138	.307
11	.288	.190	.096	.052	.130	.190	.368	.134	.099	.342		.130	.273
12	.078	.187	.228	.092	.249	.218	.237	.089	.068	.138	.130		.130
13	.285	.345	.277	.247	.351	.479	.414	.346	.294	.307	.273	.130	
14	.275	.448	.459	.328	.406	.572	.441	.371	.300	.328	.247	.205	.492
15	.172	.310	.294	.284	.215	.384	.291	.237	.272	.240	.101	.196	.335
16	.092	.284	.442	.428	.310	.318	.148	.116	.334	.135	−.065	.121	.165
17	.244	.475	.407	.396	.425	.469	.546	.272	.346	.305	.192	.233	.384
18	.684	.247	.368	.091	.237	.243	.338	.165	.186	.141	.421	.142	.275
19	.279	.374	.438	.380	.272	.404	.339	.306	.379	.353	.171	.184	.391
20	.304	.379	.624	.350	.371	.542	.343	.310	.307	.224	.212	.102	.397
21	.084	.291	.268	.326	.244	.390	.251	.295	.327	.366	.184	.167	.270
22	.105	.210	.387	.187	.614	.615	.313	.180	.201	.175	.111	.222	.397
23	.261	.434	.397	.409	.300	.368	.210	.324	.413	.352	.257	.161	.323
24	.070	.188	.447	.153	.403	.507	.197	.112	.154	.095	.046	.165	.258
25	.064	.155	.412	.166	.395	.443	.251	.090	.131	.035	−.099	.130	.233
26	.049	.223	.391	.289	.430	.559	.277	.141	.242	.203	.072	.064	.300
27	.321	.337	.120	.178	.227	.206	.361	.327	.306	.401	.500	.128	.363
28	.124	.272	.425	.209	.754	.627	.432	.163	.248	.271	.192	.245	.375
29	.215	.337	.193	.201	.207	.152	.342	.155	.226	.398	.507	.063	.284
30	.181	.160	.292	.020	.393	.374	.320	.098	.053	.032	.212	.208	.241
31	.227	.364	.415	.519	.273	.343	.227	.294	.577	.320	.240	.134	.339
32	.068	.354	.485	.348	.620	.604	.369	.195	.293	.241	−.051	.170	.353
33	.047	.252	.128	.141	.349	.244	.248	.083	.122	.481	.312	.199	.227
34	.165	.221	.137	.122	.126	.189	.136	.106	.100	.120	.185	.449	.142
35	.096	.309	.352	.201	.531	.529	.411	.216	.216	.267	.196	.225	.435
−36	−.087	.118	.185	.062	.324	.363	.123	−.030	.050	−.046	−.091	.066	.151
37	−.219	−.058	−.290	−.202	−.152	−.024	.006	.026	−.195	.117	.056	.134	.032

TABLE 2 (continued)

Correlation Matrix

	14	15	16	17	18	19	20	21	22	23	24	25
1	.275	.172	.092	.244	.684	.279	.304	.084	.105	.261	.070	.064
2	.448	.310	.284	.475	.247	.374	.379	.291	.210	.434	.188	.155
3	.459	.294	.442	.407	.368	.438	.624	.268	.387	.397	.447	.412
4	.328	.284	.428	.396	.091	.380	.350	.326	.187	.409	.153	.166
5	.406	.215	.310	.425	.237	.272	.371	.244	.614	.300	.403	.395
6	.572	.384	.318	.469	.243	.404	.542	.390	.615	.368	.507	.443
7	.441	.291	.148	.546	.338	.339	.343	.251	.313	.210	.197	.251
8	.371	.237	.116	.272	.165	.306	.310	.295	.180	.324	.112	.090
9	.300	.272	.334	.346	.186	.379	.307	.327	.201	.413	.154	.131
10	.328	.240	.135	.305	.141	.353	.224	.366	.175	.352	.095	.035
11	.247	.101	−.065	.192	.421	.171	.212	.184	.111	.257	.046	−.099
12	.205	.196	.121	.233	.142	.184	.102	.167	.222	.161	.165	.130
13	.492	.335	.165	.384	.275	.391	.397	.270	.397	.323	.258	.233
14		.391	.239	.481	.272	.507	.512	.439	.432	.375	.386	.314
15	.391		.153	.353	.129	.418	.305	.316	.230	.354	.249	.150
16	.239	.153		.411	.065	.245	.364	.196	.198	.307	.278	.332
17	.481	.353	.411		.224	.433	.410	.398	.326	.362	.291	.294
18	.272	.129	.065	.224		.327	.316	.044	.199	.312	.108	.049
19	.507	.418	.245	.433	.327		.425	.339	.229	.392	.256	.162
20	.512	.305	.364	.410	.316	.425		.322	.325	.451	.375	.369
21	.439	.316	.196	.398	.044	.339	.322		.156	.317	.212	.146
22	.432	.230	.198	.326	.199	.229	.325	.156		.265	.470	.377
23	.375	.354	.307	.362	.312	.392	.451	.317	.265		.214	.126
24	.386	.249	.278	.291	.108	.256	.375	.212	.470	.214		.368
25	.314	.150	.332	.294	.049	.162	.369	.146	.377	.126	.368	
26	.447	.188	.295	.323	.076	.256	.450	.286	.446	.237	.355	.391
27	.356	.208	.097	.306	.352	.275	.219	.305	.176	.377	.083	.023
28	.407	.203	.198	.378	.296	.294	.345	.204	.676	.244	.451	.326
29	.247	.131	.122	.275	.287	.237	.229	.180	.044	.307	.030	.018
30	.289	.098	.005	.326	.293	.148	.217	.159	.365	.086	.261	.198
31	.381	.353	.392	.379	.232	.528	.440	.365	.203	.506	.207	.197
32	.497	.277	.405	.426	.116	.354	.485	.370	.530	.298	.493	.491
33	.219	.129	.052	.155	.186	.247	.117	.118	.235	.238	.100	.026
34	.236	.168	.017	.203	.206	.142	.119	.109	.205	.179	.059	.097
35	.477	.341	.226	.373	.174	.256	.375	.306	.545	.280	.391	.291
−36	.219	.063	.207	.204	−.086	.059	.205	.082	.357	−.013	.332	.386
37	.080	.029	−.372	−.161	−.193	−.037	−.249	−.013	.076	−.164	−.014	−.147

TABLE 2 (continued)

Correlation Matrix

	26	27	28	29	30	31	32	33	34	35	—36	37
1	.049	.321	.124	.215	.181	.227	.068	.047	.165	.096	−.087	−.219
2	.223	.337	.272	.337	.160	.364	.354	.252	.221	.309	.118	−.058
3	.391	.120	.425	.193	.292	.415	.485	.128	.137	.352	.185	−.290
4	.289	.178	.209	.201	.020	.519	.348	.141	.122	.201	.062	−.202
5	.430	.227	.754	.207	.393	.273	.620	.349	.126	.531	.324	−.152
6	.559	.206	.627	.152	.374	.343	.604	.244	.189	.529	.363	−.024
7	.277	.361	.432	.342	.320	.227	.369	.248	.136	.411	.123	.006
8	.141	.327	.163	.155	.098	.294	.195	.083	.106	.216	−.030	.026
9	.242	.306	.248	.226	.053	.577	.293	.122	.100	.216	.050	−.195
10	.203	.401	.271	.398	.032	.320	.241	.481	.120	.267	−.046	.117
11	.072	.500	.192	.507	.212	.240	−.051	.312	.185	.196	−.091	.056
12	.064	.128	.245	.063	.208	.134	.170	.199	.449	.225	.066	.134
13	.300	.363	.375	.284	.241	.339	.353	.227	.142	.435	.151	.032
14	.447	.356	.407	.247	.289	.381	.497	.219	.236	.477	.219	.080
15	.188	.208	.203	.131	.098	.353	.277	.129	.168	.341	.063	.029
16	.295	.097	.198	.122	.005	.392	.405	.052	.017	.226	.207	−.372
17	.323	.306	.378	.275	.326	.379	.426	.155	.203	.373	.204	−.161
18	.076	.352	.296	.287	.293	.232	.116	.186	.206	.174	−.086	−.193
19	.256	.275	.294	.237	.148	.528	.354	.247	.142	.256	.059	−.037
20	.450	.219	.345	.229	.217	.440	.485	.117	.119	.375	.205	−.249
21	.286	.305	.204	.180	.159	.365	.370	.118	.109	.306	.082	−.013
22	.446	.176	.676	.044	.365	.203	.530	.235	.205	.545	.357	−.076
23	.237	.377	.244	.307	.086	.506	.298	.238	.179	.280	−.013	−.164
24	.355	.083	.451	.030	.261	.207	.493	.100	.059	.391	.332	−.014
25	.391	.023	.326	.018	.198	.197	.491	.026	.097	.291	.386	−.147
26		.102	.497	.118	.346	.253	.532	.243	.073	.386	.356	−.006
27	.102		.180	.513	.229	.322	.130	.293	.192	.191	−.085	.093
28	.497	.180		.197	.481	.210	.544	.392	.132	.533	.374	.003
29	.118	.513	.197		.103	.335	.123	.317	.116	.196	−.043	−.082
30	.346	.229	.481	.103		.039	.330	.199	.111	.307	.290	.117
31	.253	.322	.210	.335	.039		.366	.041	.146	.238	.089	−.187
32	.532	.130	.544	.123	.330	.366		.196	.103	.525	.492	−.066
33	.243	.293	.392	.317	.199	.041	.196		.080	.277	−.001	.124
34	.073	.192	.132	.116	.111	.146	.103	.080		.173	−.023	.088
35	.386	.191	.533	.196	.307	.238	.525	.277	.173		.233	.096
−36	.356	−.085	.374	−.043	.290	.089	.492	−.001	−.023	.233		.032
37	−.006	.093	.003	−.082	.117	−.187	−.066	.124	.088	.096	.032	

TABLE 3

Centroid Factor Matrix

	I	*II*	*III*	*IV*	*V*	*VI*	*VII*	*VIII*	*IX*	*X*	*XI*	h^2
1	.369	.283	−.146	.432	.342	−.109	−.141	−.170	−.122	−.105	−.094	.637
2	.570	.185	.095	−.036	−.085	−.043	−.096	.133	−.059	−.117	−.069	.427
3	.652	−.193	.196	.244	.224	.125	−.028	.142	−.157	−.083	.116	.692
4	.520	.171	.449	−.258	.100	.265	−.114	−.133	.037	.164	−.125	.723
5	.684	−.341	−.221	−.218	−.035	.180	−.075	−.084	−.064	−.053	−.064	.738
6	.754	−.295	−.011	.096	−.106	−.053	.101	−.015	−.014	.045	−.009	.692
7	.606	.089	−.193	−.092	−.049	−.209	−.216	−.088	−.065	−.082	.099	.542
8	.408	.217	.101	.101	−.079	−.095	.096	−.099	−.022	.069	−.135	.292
9	.531	.241	.340	−.234	.155	.239	−.070	−.307	.107	.152	−.065	.729
10	.479	.326	−.071	−.344	−.212	.085	.227	.136	−.183	−.060	−.108	.630
11	.363	.392	−.415	−.072	.198	−.150	.202	.105	.082	.028	.179	.616
12	.333	.052	−.180	.160	−.236	.266	−.268	.270	.146	.106	.138	.494
13	.603	.083	−.073	.064	−.100	−.187	.119	−.194	.110	−.100	−.104	.510
14	.720	.015	.073	.132	−.177	−.210	.076	.012	−.043	.074	−.053	.633
15	.475	.134	.155	.142	−.257	.022	.072	−.072	.056	−.118	.097	.382
16	.428	−.138	.378	−.127	.153	.106	−.150	.175	.050	−.179	−.087	.491
17	.667	.056	.108	−.069	−.096	−.132	−.278	.044	−.048	−.059	.114	.576
18	.446	.249	−.336	.354	.412	.058	−.123	−.144	−.202	−.126	−.019	.765
19	.601	.220	.208	.127	−.117	.099	.083	−.092	−.201	−.101	.184	.592
20	.661	−.099	.227	.179	.196	−.132	.121	.099	−.102	−.057	−.034	.625
21	.492	.124	.213	−.092	−.172	−.147	.087	.051	−.054	.181	.053	.411
22	.604	−.404	−.227	.065	−.107	.132	.096	−.133	.165	.063	−.126	.687
23	.580	.236	.174	.038	.099	.151	.176	.087	.017	−.051	−.107	.509
24	.492	−.388	.054	.117	−.037	.038	.112	.050	.083	−.018	.109	.446
25	.426	−.427	.130	.058	.032	−.068	−.126	.087	.085	−.072	−.064	.430
26	.555	−.358	.061	−.092	.047	−.101	.123	.027	−.078	.189	−.080	.525
27	.484	.448	−.205	−.085	.094	−.175	.082	.046	.105	.038	−.047	.547
28	.671	−.378	−.343	−.124	−.049	.172	.028	−.160	−.107	.067	.073	.806
29	.418	.371	−.178	−.286	.219	−.147	.086	.164	.066	−.169	.078	.569
30	.421	−.217	−.301	.031	.024	−.161	−.133	−.054	−.096	.223	.175	.453
31	.587	.229	.372	−.039	.156	.120	.102	−.089	.181	−.056	.131	.647
32	.676	−.424	.158	−.121	−.110	−.043	−.071	−.038	−.026	−.081	−.075	.710
33	.381	.100	−.344	−.259	−.077	.161	.167	.152	−.196	−.025	−.095	.471
34	.291	.179	−.160	.241	−.144	.142	−.209	.225	.227	.191	−.090	.432
35	.617	−.202	−.142	−.028	−.207	−.017	.080	−.045	.095	−.101	−.042	.515
36	.285	−.519	.061	−.096	−.037	−.152	−.103	−.040	.116	.052	.072	.422
37	−.091	.062	−.283	.016	−.497	−.129	.189	−.056	.143	.276	.041	.493

TABLE 4

Direction Cosines λ_{mp} of Primary Reference Vectors Λ_p

	P	*N*	*W*	*V*	*S*	*M*	*I*	*D*	9	10	11
I	.177	.109	.222	.318	.124	.096	.234	.225	.079	.066	.052
II	.234	.130	.190	−.565	.203	.110	.124	−.369	.130	.039	.013
III	−.246	−.037	.080	−.481	.343	−.138	.277	.462	−.146	−.043	−.082
IV	−.341	.567	−.071	−.176	−.333	.354	.448	.000	−.242	.067	−.121
V	.017	.367	−.552	−.067	.294	−.243	−.476	.427	.351	.048	−.158
VI	.145	.164	−.536	.043	.441	.354	−.101	−.144	−.240	−.436	.278
VII	.430	.024	−.333	.164	−.048	−.303	.596	.004	.269	.219	.185
VIII	.448	−.086	−.198	−.480	−.452	.530	−.201	.609	.178	−.040	.160
IX	−.379	−.681	−.334	.231	.275	.377	.083	−.149	.639	−.107	−.412
X	−.055	−.123	−.157	−.002	.387	.367	−.114	.043	−.112	.857	.068
XI	−.428	.000	.159	−.028	−.045	−.004	.004	−.030	.438	.000	.800

TABLE 5

Cosines of Angular Separations Between Reference Vectors Λ_p

	P	*N*	*W*	*V*	*S*	*M*	*I*	*D*	9	10	11
P	1.000										
N	.132	1.000									
W	−.164	−.029	1.000								
V	−.112	−.250	−.090	1.000							
S	−.162	−.179	−.336	.091	1.000						
M	−.051	−.155	−.238	−.214	−.025	1.000					
I	−.034	.083	.215	.033	−.144	−.061	1.000				
D	.168	.136	−.225	−.302	−.125	.044	−.191	.999			
9	−.094	−.447	−.268	.125	.083	−.003	−.065	.083	1.001		
10	.016	−.026	.057	.015	.111	.061	.092	.093	.009	1.000	
11	.073	.192	.115	−.072	−.107	.007	.023	.000	.083	.007	1.001

TABLE 6

Correlations of Primary Factors

	P	*N*	*W*	*V*	*S*	*M*	*I*	*D*
P	1.000							
N	.108	1.000						
W	.312	.389	1.000					
V	.144	.294	.314	1.000				
S	.258	.263	.475	.160	1.000			
M	.200	.364	.443	.357	.261	1.000		
I	.006	−.105	−.108	−.013	.108	.003	1.000	
D	−.029	−.026	.257	.327	.234	.120	.184	1.000

TABLE 7

First Factor Residuals

	P	*N*	*W*	*V*	*S*	*M*	*I*	*D*
P	.911							
N	−.007	.851						
W	.093	.105	.458					
V	−.012	.093	−.072	.725				
S	.078	.030	.031	−.156	.636			
M	.025	.136	.009	.048	−.095	.652		
I	.006	−.105	−.107	−.013	.108	.003	1.000	
D	−.124	−.148	.022	.160	.042	−.068	.184	.898

TABLE 8

Saturations of Primaries in Second-Order General Factor

P	*N*	*W*	*V*	*S*	*M*	*I*	*D*
.298	.386	.736	.524	.603	.590	.000	.319